# Measurements of the effect of molecular diffusivity in turbulent diffusion

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Townsend (1954) has shown that turbulent vorticity may rotate and strain a diffusion wake, thereby increasing the contribution of molecular diffusion to the total mean dispersion over short diffusion times. To test whether any such effect occurs at longer diffusion times, the lateral dispersion of both helium and of carbon dioxide in air were measured downstream from a continuous point source in the turbulence produced by a grid in a wind tunnel. The data show that, for long diffusion times, accelerated molecular diffusion is negligible, so that molecular diffusion makes only an independent contribution to the total dispersion.

## 1. Introduction

The turbulent diffusion of quantities such as mass and heat can be characterized by the mean-square displacement of fluid particles from their original position as a function of time. Taylor (1922) has shown that the mean-square displacement $\overline{Y^2}$, in a given direction, and due to homogeneous turbulence is

$$\overline{Y^2} = 2\overline{v^2}\int_0^t\int_0^{t'} R\,dt'\,dt'', \qquad (1)$$

where the Lagrangian correlation coefficient $\overline{v^2}R(t''-t_0) = \overline{v_0 v_{t''}}$ is an average of the product of the turbulent velocities of a fluid particle at time zero and at a later time $t''$. Experiment has shown that lateral concentration profiles downstream from a continuous point source are of Gaussian form (Batchelor & Townsend (1956) discuss some theoretical reasons for this), i.e.

$$C = C_0 \exp(-r^2/2D^2), \qquad (2)$$

where $C$ is the concentration at radius $r$ from the diffusion wake centre, and $C_0$ is the concentration at the wake centre. The total mean lateral dispersion $D^2$ has often been assumed to be merely the sum of contributions due to the turbulent and molecular diffusion, in which case

$$D^2 = \overline{Y^2} + 2\alpha t, \qquad (3)$$

where $\alpha$ is the molecular diffusion coefficient.

Townsend (1954) has shown that for short diffusion times

$$D^2 = \overline{Y^2} + 2\alpha t(1 + \tfrac{14}{15}\overline{\omega^2}t^2 + \text{higher order terms}), \qquad (4)$$

where $\boldsymbol{\omega}$ is the vorticity of the turbulent motion, and the higher order terms constitute the rest of an alternating series of very complex terms which are negligible for small times. The difference between equations (3) and (4) arises from consideration of the rotation and strain of the instantaneous diffusion wake caused by the turbulent vorticity, which acts to increase the rate of molecular diffusion and thereby increases the lateral dispersion observed by concentration measurements. Townsend (1954), and Batchelor & Townsend (1956), have suggested that an effect such as that expressed by the terms in $t^3$, $t^5$, etc., in equation (4) might hold for longer diffusion times than those investigated by Townsend (1954); in that event, interpretation of diffusion experiments would be affected by the value of the molecular diffusivity. If the total dispersion is in fact influenced by interaction between molecular conduction and turbulent convection at large values of $t$, then many previous experiments (such as the experimental comparison of Eulerian and Lagrangian correlation coefficients by Mickelsen (1955)) would be in serious doubt.

As noted by Townsend (1954), equation (4) may be expressed in terms of the Prandtl or Schmidt numbers when applied to the decaying turbulent field downstream of grids; for, on neglecting the higher order terms, we have

$$\frac{D^2}{x_0 M} = \frac{\overline{Y^2}}{x_0 M} + \frac{2\alpha t}{x_0 M} + \frac{14}{15A}\frac{\alpha}{\nu}\left(\frac{x-x_0}{x_0}\right)^3\left(\frac{x_0}{x}\right)^2 \tag{5}$$

(in which the normalization is due to Batchelor (1952)), where $x_0$ is the distance from the virtual source of turbulence to the source of diffusion, $x-x_0$ is the distance downstream from the source of diffusion, $M$ is the mesh length of the turbulence generating grid, $A$ is approximately 135 for Townsend's (1954) turbulent field, and $\nu$ is the kinematic viscosity of air. From equation (5), it is evident that the value of the Prandtl number $\nu/\alpha$ for the diffusion of heat in air, and of the Schmidt number $\nu/\alpha$ for the diffusion of another gas in air, should markedly affect the total dispersion at the small values of $t$ for which the relation (5) is a valid approximation. The present experiment was performed to test the suggestion that $D^2 - 2\alpha t$ would depend on $\nu/\alpha$ at longer diffusion times than those investigated by Townsend (1954). The experiment essentially consisted of comparing the mean lateral dispersion $D^2$ of two gases of widely different molecular diffusivities, and hence Schmidt numbers, in the same flowing turbulent field previously used by Townsend (1954).

## 2. The experiment

The experiment was carried out in the wind tunnel in the Cavendish Laboratory in which Townsend (1954) originally observed the accelerated molecular diffusion of heat downstream from a line source. The wind tunnel, injector, and sampling apparatus of the present experiment are shown in figure 1. Helium and carbon dioxide were used for the diffusing gas because of (*a*) the wide difference in their molecular diffusivities (for helium $\alpha = 0{\cdot}725\,\text{cm}^2/\text{s}$; for carbon dioxide $\alpha = 0{\cdot}167\,\text{cm}^2/\text{s}$; both at 80 °F and 1 atm.), and (*b*) the convenience in measuring their concentrations with the katharometer (thermal conductivity bridge). The gas was injected into the turbulent air stream through a small tube of 0·8 mm outside diameter (2 mm for the large mesh grid data), as shown in figure 1. Helium was always injected at stream velocity, but it was necessary to inject the carbon dioxide at four or five times the stream velocity in order to obtain concentrations high enough for accurate measurement. Several concentration profiles were measured at $(x-x_0)/x_0 = 0{\cdot}1$, with the high injection velocity, and with injection at stream velocity. The negligible difference in these profiles indicated that the injector, even with the high injection velocity used for carbon dioxide, was closely simulating a point source.

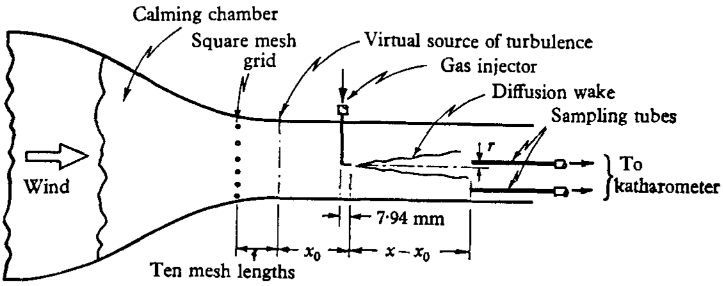

FIGURE 1. Experimental apparatus.

Concentrations of gas downstream from the injector were measured by continuously drawing samples through the katharometer from both the diffusion wake and from the pure air outside the wake. The sampling tubes were 0·6 mm inside diameter, and the sampling rate was at stream velocity. The concentration profiles were found to be as in equation (2), which enabled the mean dispersion $D^2$ to be determined graphically.

## 3. Results

Concentration measurements were made at a number of axial stations downstream from the injector, using two widely different grid meshes at a fixed wind speed of 640 cm/sec. The total contribution $D^2 - 2\alpha t$ to the dispersion due directly and indirectly to the turbulence was determined from these data, and is shown in figure 2. The dispersion and axial distance have been normalized as in (5) to account for the decrease in turbulence intensity and increase in turbulence scale with distance downstream. Estimates indicated that the turbulent wake from the cylindrical injector was small compared with the turbulence generated by the grid, and this is borne out by the good agreement between the gas diffusion data and Townsend's (1954) data for diffusion of heat from a line source which was small enough to leave no wake.

To indicate what trends would be expected if accelerated molecular diffusion were appreciable, values of $(D^2 - 2\alpha t)/x_0 M$ for helium, carbon dioxide, and heat were calculated from (5) (which is of course valid only for small $t$), and are shown as dashed lines in figure 2. These calculations were made by using Townsend's (1954) value for turbulence intensity at $(x-x_0)/x_0 = 0{\cdot}1$, and by assuming that Townsend's data for longer diffusion times (shown by the solid line) represented the turbulent mean-square displacement $\overline{Y^2}/x_0 M$.

It is evident from figure 2 that, at least within experimental error, equation (3) adequately describes the total dispersion for the diffusion times and turbulent field used in the present experiment and that accelerated molecular diffusion does not make a significant contribution. Whether these results may be generalized to all situations is still open to some question which might be answered by further experiment. It is possible that Townsend's accelerated molecular diffusion does not obtain in cases where the diffusion wake has been sufficiently broken up, as it is when the diffusion time is large.

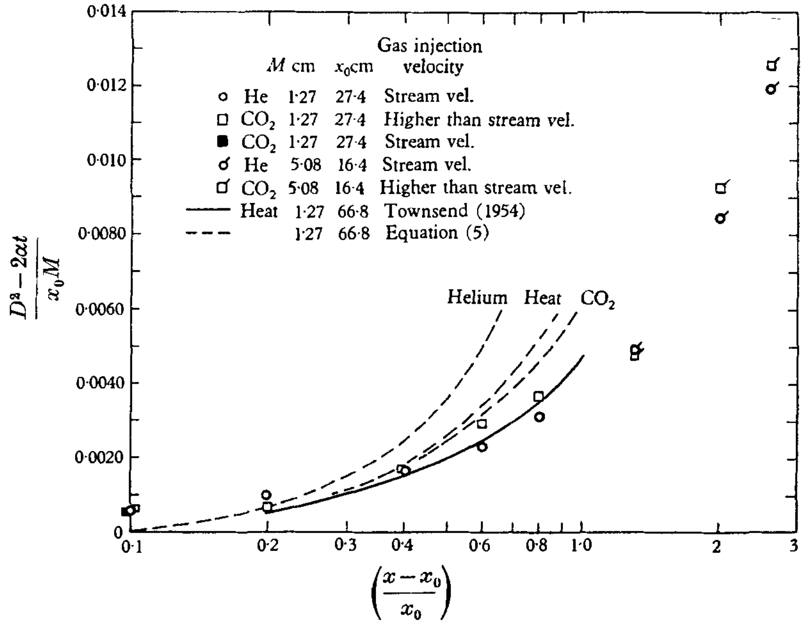

FIGURE 2. Comparison of turbulent dispersion of helium and carbon dioxide.

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